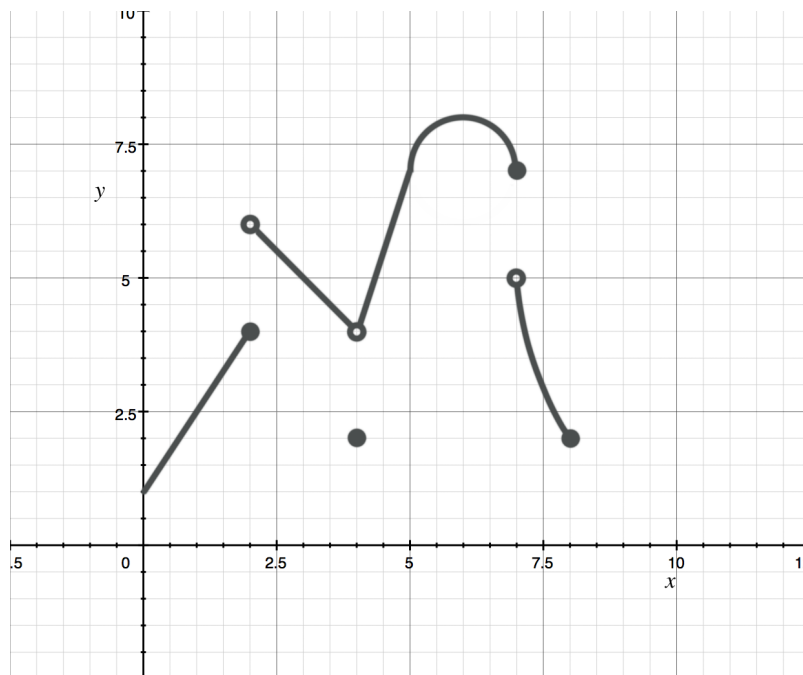


**SECTION I, Part A**

**Time - 60 Minutes**

**Number of questions - 30**

**A CALCULATOR MAY NOT BE USED IN THIS PART OF THE EXAMINATION.**



1. The graph of the function  $f$  is shown above. Which of the following statements is false?

(A)  $\lim_{x \rightarrow 2} f(x)$  exists.

(B)  $\lim_{x \rightarrow 4} f(x)$  exists.

(C)  $\lim_{x \rightarrow 5} f(x)$  exists.

(D) The function  $f$  is discontinuous at  $x = 4$ .

2. What is  $\lim_{h \rightarrow 0} \frac{25 \left(\frac{1}{5} + h\right)^5 - 25 \left(\frac{1}{5}\right)^5}{h}$ ?

(A) 0

(B)  $\frac{1}{5}$

(C) 1

(D) The limit does not exist.

3. For what value of  $k$  is the function  $f$  continuous at  $x = 4$ ?

$$f(x) = \begin{cases} \frac{\sqrt{3x+4} - \sqrt{2x+8}}{x-4} & x \neq 4 \\ k & x = 4 \end{cases}$$

(A) 0

(B)  $\frac{1}{16}$

(C)  $\frac{1}{8}$

(D)  $\frac{1}{4}$

4.  $\lim_{x \rightarrow 0} \frac{\cos(2x) - \sin(2x) - 1}{x + \sin x}$

- (A)  $-1$
- (B)  $0$
- (C)  $1$
- (D) Nonexistent

5.  $\frac{d}{dx}(x \cos x)$

- (A)  $-\sin x$
- (B)  $\cos x - \sin x$
- (C)  $-\cos x + x \sin x$
- (D)  $\cos x - x \sin x$

6. If  $f(x) = x^2 - \frac{4}{x} + 2\sqrt{2x-1}$ , then find  $f'(1)$ .

- (A)  $-1$
- (B)  $0$
- (C)  $2$
- (D)  $8$

7. If  $y = (\sqrt{x} + \sin x)^4$ , then find  $y'$ .

(A)  $4(\sqrt{x} + \sin x)^3$

(B)  $4\left(\frac{1}{2\sqrt{x}} + \cos x\right)^3$

(C)  $4\left(\frac{1}{2\sqrt{x}} + \cos x\right)$

(D)  $4\left(\frac{1}{2\sqrt{x}} + \cos x\right)(\sqrt{x} + \sin x)^3$

8. If  $f(x) = \frac{x}{\tan x}$ , calculate  $f'\left(\frac{\pi}{3}\right)$ .

(A)  $-\frac{\sqrt{3}}{3}$

(B)  $\sqrt{3}$

(C)  $\frac{3\sqrt{3} - 4\pi}{9}$

(D)  $\frac{\sqrt{3}(3 + 2\pi)}{9}$

9. Let  $f$  be a function such that  $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = 9$ . Which of the following must be true?

- I.  $f$  is continuous at  $x = 3$ .
- II.  $f$  is differentiable at  $x = 3$ .
- III.  $f'$  is continuous at  $x = 3$ .

- (A) I only
- (B) II only
- (C) I and II
- (D) II and III

10. What is the average value of  $y = x^2\sqrt{3x^3 + 1}$  on the interval  $[0,2]$ ?

- (A)  $\frac{124}{27}$
- (B)  $\frac{248}{27}$
- (C)  $\frac{124}{9}$
- (D)  $\frac{124}{3}$

11. Let  $f$  be the function given by  $f(x) = x^3 - 75x + 10$ . On which of the following intervals is the function  $f$  decreasing?

(A)  $(-\infty, -5] \cup [5, \infty)$

(B)  $[-5, 5]$

(C)  $[0, 5]$

(D)  $[0, 5\sqrt{3}]$

12.  $\int \csc^2 x \, dx$

(A)  $\tan x + C$

(B)  $\cot x + C$

(C)  $-\cot x + C$

(D)  $\frac{1}{3} \csc^3 x + C$

13. A particle moves along the  $x$ -axis. The velocity of the particle at time  $t$  is  $9t^2 - 2t - 7$ . What is the total distance traveled by the particle from time  $t = 0$  to  $t = 2$ ?

(A) 16

(B) 18

(C) 10

(D) 13

t (hours)	5	8	11	15	19
R(t) (liters/hour)	6.5	5.0	4.0	3.5	2.0

14. A tank contains 30 liters of water at time  $t = 5$  hours. Water is being pumped into the tank at a rate  $R(t)$  where  $R(t)$  is measured in liters per hour, and  $t$  is measured in hours. Selected values of  $R(t)$  are given in the table above. Using a right Riemann sum with four subintervals and data from the table, what is the approximation of the number of liters of water that are in the tank at time  $t = 19$  hours?

- (A) 49
- (B) 73.5
- (C) 79
- (D) 88

15. Find the area between the curves  $y = 3x^2 + x - 2$  and  $y = x + 1$ .

- (A)  $\frac{2}{3}$
- (B) 2
- (C) 4
- (D) 6

16. Let  $f$  be the function defined by  $f(x) = -\sqrt{|x|}$  for all values of  $x$ . Which of the following statements is true?

- (A)  $f$  is differentiable at  $x = 0$ .
- (B)  $f$  is not continuous at  $x = 0$ .
- (C)  $\lim_{x \rightarrow 0} f(x) \neq 0$
- (D)  $f$  is continuous but not differentiable at  $x = 0$ .

17. Using the substitution  $u = 3x + 4$ ,  $\int_0^2 3(3x + 4)^4 dx$  is equal to which of the following?

- (A)  $\int_4^{10} u^4 du$
- (B)  $\frac{1}{3} \int_0^2 u^4 du$
- (C)  $3 \int_4^{10} u^4 du$
- (D)  $\int_0^2 u^4 du$

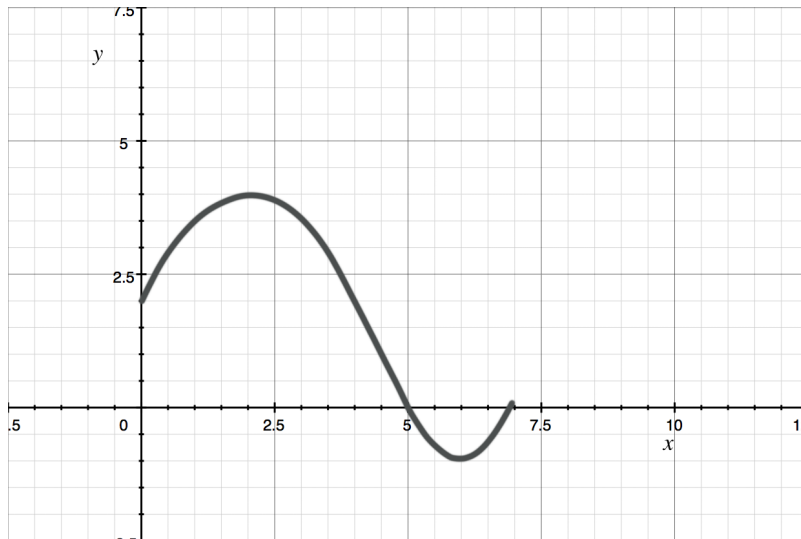
18. The function  $f$  is defined by  $f(x) = \begin{cases} 2x + 4 & x < 1 \\ -3x + 9 & x \geq 1 \end{cases}$ . What is the value of

$$\int_0^3 f(x) \, dx?$$

- (A)  $-1$
- (B)  $11$
- (C)  $21$
- (D)  $26$

19. If  $f(x) = 2x^3$  and  $g(x) = 2x + 3$ , then find the derivative of  $f(g(x))$  at  $x = 1$ .

- (A)  $25$
- (B)  $75$
- (C)  $150$
- (D)  $300$

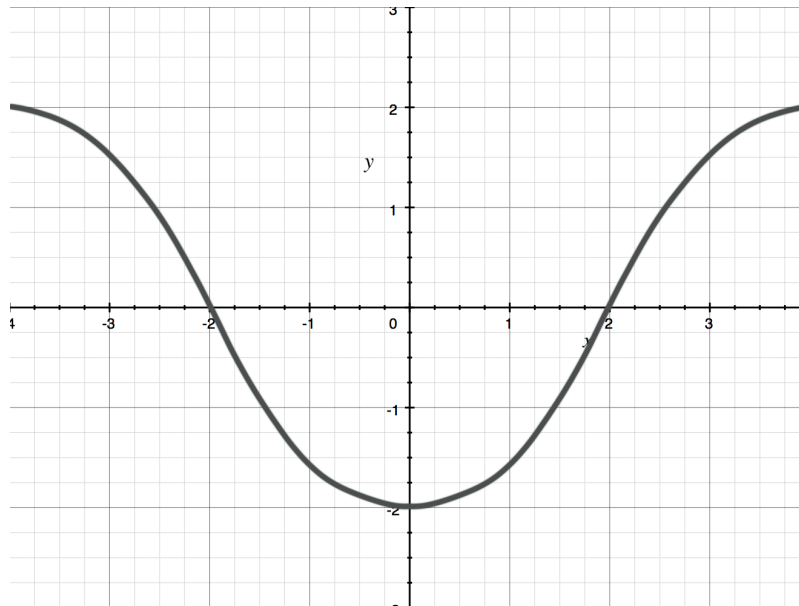


20. The graph of a differentiable function  $f$  is shown above. If  $g(x) = \int_0^x f(t) dt$ , which of the following statements is true?

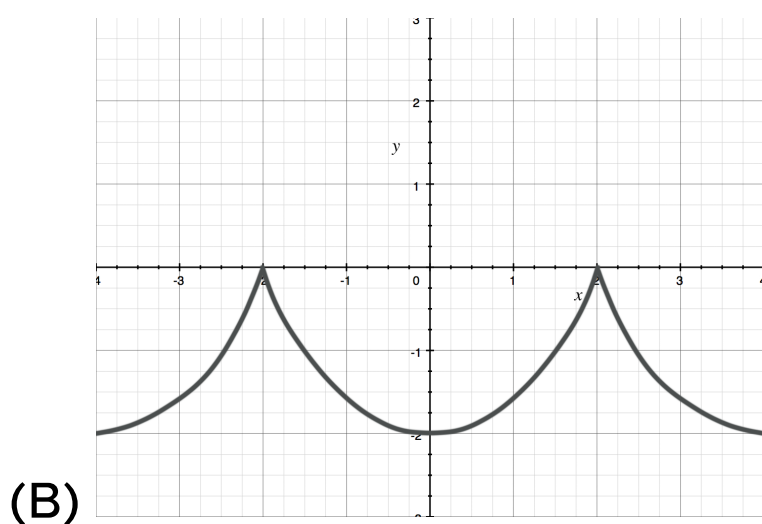
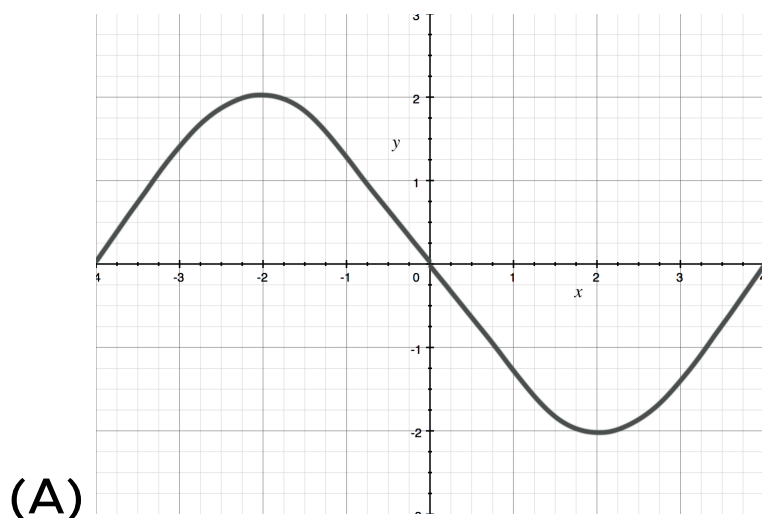
- (A)  $g(5) < g'(5) < g''(5)$
- (B)  $g(5) < g''(5) < g'(5)$
- (C)  $g'(5) < g(5) < g''(5)$
- (D)  $g''(5) < g'(5) < g(5)$

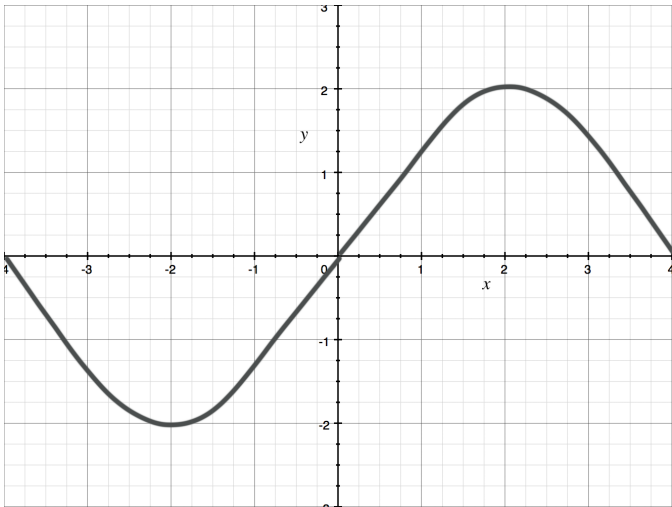
21. A particle moves along the  $x$ -axis so that at any time  $t \geq 0$  its position is given by  $x(t) = t^2 e^{-3t}$ . For what values of  $t$  the particle is at rest?

- (A) 0 only
- (B)  $\frac{2}{3}$  only
- (C) 0 and  $\frac{2}{3}$
- (D) 1 only

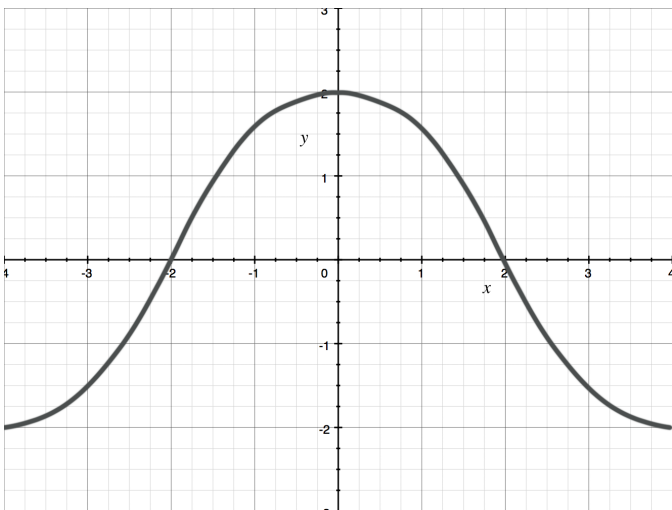


22. The figure above shows the graph of  $f$ . If  $f(x) = \int_0^x g(t) dt$ , which of the following could be the graph of  $y = g(x)$ ?





(C)



(D)

23. The function  $f$  is defined by  $f(x) = \frac{x^2}{2x + 1}$ . What point  $(x, y)$  on the graph of  $f$  has the property that the line tangent to  $f$  at  $(x, y)$  has slope 0?

- (A)  $(0, 0)$  only
- (B)  $(-1, -1)$  only
- (C)  $(0, 0)$  and  $(-1, -1)$
- (D)  $(-1, -2)$  only

24. Let  $f(x) = x^5 - 1$  and let  $g$  be the inverse of  $f$ . Given that  $f(1) = 0$ , what is the value of  $g'(0)$ ?

(A)  $-\frac{1}{4}$

(B) 0

(C)  $\frac{1}{5}$

(D) 1

25. If the graph of  $y = \frac{ax^2 + b}{c - x^2}$  has a horizontal asymptote  $y = 5$  and vertical asymptotes  $x = 2$  and  $x = -2$ , then find  $a + c$ .

(A)  $-9$

(B)  $-1$

(C) 0

(D) 1

26. What is the absolute minimum of the function  $f$  given by  $f(x) = xe^{2x}$ ?

(A)  $-\frac{1}{2}$

(B)  $-\frac{1}{2e}$

(C)  $-\frac{e}{2}$

(D) 0

t	0	1
f(t)	3	7

27. Let  $y = f(t)$  be a solution to the differential equation  $\frac{dy}{dt} = k$ , where  $k$  is a constant. The values of  $f$  for selected values of  $t$  are given in the table above. Which of the following is an expression for  $f(t)$ ?

(A)  $3e^{t \ln \frac{7}{3}}$

(B)  $2t^2 + 3$

(C)  $e^{t \ln 7} + 3$

(D)  $4t + 3$

28. Let  $f$  be the function given by  $f(x) = (x^2 + 1)e^{-kx}$ , where  $k$  is a constant. For what value of  $k$  does  $g$  have critical points at  $x = 1$ ?

(A)  $-1$

(B)  $0$

(C)  $1$

(D)  $\ln 1$

29. Which of the following is the solution to the differential equation  $\frac{dy}{dx} = xe^{-y}$  with the initial condition  $y(0) = 1$ ?

(A)  $y = e^{\frac{x^2}{2}}$

(B)  $y = \ln\left(\frac{x^2}{2}\right)$

(C)  $y = \ln\left(\frac{x^2}{2} + \frac{1}{2}\right)$

(D)  $y = e^{\frac{x^2}{2} + \frac{1}{2}}$

30. For  $t \geq 0$ , the position of a particle moving along the  $x$ -axis is given by  $x(t) = \cos t + \sin t$ . What is the acceleration of the particle at the point where the velocity is first equal to 0?

(A)  $-\sqrt{2}$

(B)  $-1$

(C)  $0$

(D)  $1$

**END OF PART A, SECTION I**

**SECTION I, Part B****Time - 45 Minutes****Number of questions - 15****A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION.**

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31. If  $(3x^2y^2 + 2x)\frac{dy}{dx} = 2y - 2xy^3$ , what is the value of  $\frac{d^2y}{dx^2}$  at the point  $(-1,1)$ ?

(A)  $-4$

(B)  $0$

(C)  $4$

(D)  $30$

32. Let  $f$  be the function given by  $f(x) = 5e^{3x^2}$ . For what value of  $x$  is the slope of the line tangent to the graph of  $f$  at  $(x, f(x))$  equal to 1?

(A)  $-0.578$

(B)  $0.028$

(C)  $0.033$

(D)  $0.097$

33. Let  $f$  be a continuous function of the closed interval  $[-2,7]$ . If  $f(-2) = -1$  and  $f(7) = 5$ , then the Intermediate Value Theorem guarantees that

(A)  $f(0) = 0$ .

(B)  $f'(x) = 5$  has at least one solution in the open interval  $(-2,7)$ .

(C)  $f(x) = 0$  for at least one  $x$  in the open interval  $(-2,7)$ .

(D)  $f$  attains a maximum on the open interval  $(-2,7)$ .

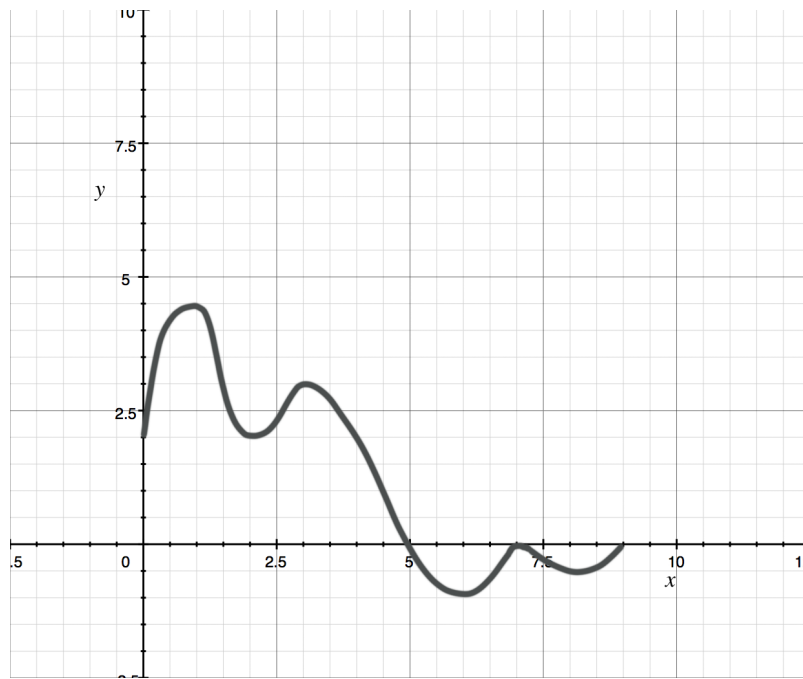
34. The graph of  $y = 3 \ln(\sec x)$  crosses the  $x$ -axis at one point in the interval  $[6,7]$ . What is the slope of the graph at this point?

(A)  $-6.283$

(B)  $-3.142$

(C)  $0$

(D)  $3$



35. The function  $f$  is defined on the closed interval  $[0,9]$ . The graph of its derivative  $f'$  is shown above. Which of the following statements must be true?

- I.  $f$  has six inflection points.
- II.  $f$  has a relative minimum at  $x = 5$ .
- III. The graph of  $f$  is concave down for  $3 < x < 6$ .

- (A) I only
- (B) II only
- (C) III only
- (D) I and III

36. Given the function defined by  $f(x) = 4x^5 - 40x^3$ , find all values of  $x$  for which the graph of  $f$  is concave up.

(A)  $x > 0$

(B)  $-\sqrt{3} < x < 0$  or  $x > \sqrt{3}$

(C)  $-3 < x < 0$  or  $x > 3$

(D)  $x > \sqrt{3}$

37. If  $f$  is a continuous function and if  $F'(x) = f(x)$  for all real numbers  $x$ , then

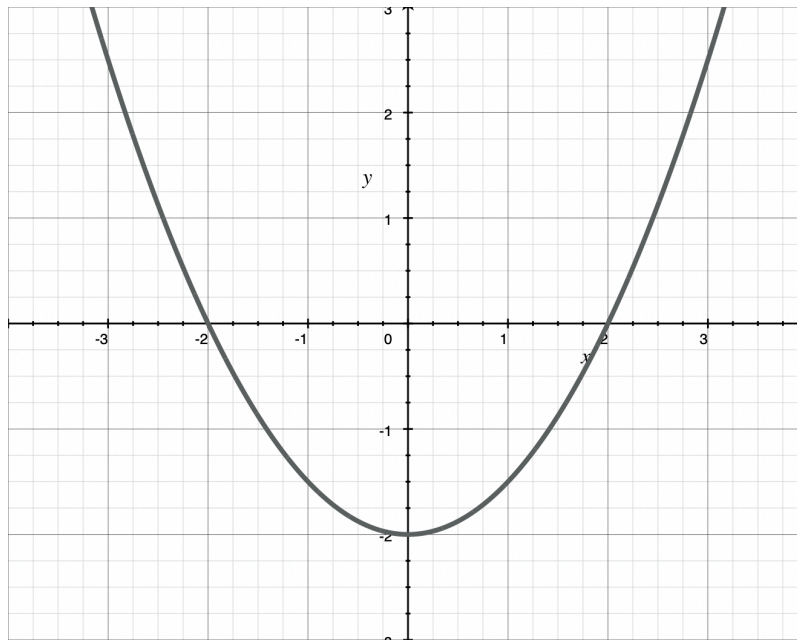
$$\int_2^5 f(2x) dx =$$

(A)  $2F(5) - 2F(2)$

(B)  $\frac{1}{2}F(5) - \frac{1}{2}F(2)$

(C)  $\frac{1}{2}F(10) - \frac{1}{2}F(4)$

(D)  $F(10) - F(4)$



38. The graph of  $f''$ , the second derivative of  $f$ , is shown above for  $-3 \leq x \leq 3$ . What are all intervals on which the graph of the function  $f$  is concave down?

- (A)  $-3 < x < 0$
- (B)  $0 < x < 3$
- (C)  $-3 < x < -2$  and  $2 < x < 3$
- (D)  $-2 < x < 2$

39. If the base  $b$  of a triangle is increasing at a rate of 2 centimeters per minute while its height  $h$  is increasing at a rate of 2 centimeters per minute, which of the following must be true about the area  $A$  of the triangle?
- (A)  $A$  is always increasing.
- (B)  $A$  is always decreasing.
- (C)  $A$  is decreasing only when  $h < -b$ .
- (D)  $A$  is decreasing only when  $h > -b$ .
40. A particle moves along a line so that its acceleration for  $t \geq 0$  is given by  $a(t) = t^2 + \cos t$ . If the particle's velocity at  $t = 0$  is  $-1$ , for what value of  $t$  will the velocity of the particle be zero?
- (A)  $-0.88$
- (B)  $0.71$
- (C)  $0.78$
- (D)  $0.88$

41. Let  $f$  be a function such that  $\int_5^{11} f(3x) dx = 15$ . Which of the following must be true?

(A)  $\int_{15}^{33} f(t) dt = 5$

(B)  $\int_{15}^{33} f(t) dt = 45$

(C)  $\int_5^{11} f(t) dt = 5$

(D)  $\int_5^{11} f(t) dt = 45$

$x$	-3	0	2	5	7
$f(x)$	-3	2	7	1	4

42. Let  $f$  be a polynomial function with values of  $f'(x)$  at selected values of  $x$  given in the table above. Which of the following must be true for  $-3 < x < 7$ ?

(A)  $f$  has at least two critical points.

(B)  $f$  is concave down.

(C)  $f$  has two inflection points.

(D) The range of  $f$  is 7.

43. Find the area between the curves  $y = 3x^2 + x - 2$  and  $y = x + 3$ .

(A) 4.304

(B) 6.545

(C) 6.873

(D) 8.607

44. Find the volume of the solid obtained by rotating the region enclosed by the curves  $y = x^2 + 3$  and  $y = x + 3$  about the  $x$ -axis.

(A) 0.105

(B) 3.560

(C) 0.524

(D) 1.133

45. A railroad track and a road cross at right angles. An observer stands on the road 90 meters south of the crossing and watches an eastbound train traveling at 65 meters/second. At how many meters/second is the train moving away from the observer 8 seconds after it passes through the intersection?
- (A) 0.985
- (B) 38.057
- (C) 64.048
- (D) 65.967

**END OF PART B, SECTION I**

**SECTION II, PART A**

**Time - 30 Minutes**

**Number of problems - 2**

**A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS.**

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1. Men enter a line for a cableway at a rate modeled by the function  $f$  such that

$$f(t) = \begin{cases} \frac{7}{1,250}t^2 \left(\frac{150-t}{150}\right)^6 & 0 \leq t \leq 150 \\ 0 & t > 150 \end{cases}$$

where  $f(t)$  is measured in men per minute and  $t$  is measured in minutes. As men get on the cableway, they exit the line at a constant rate of 0.4 men per minute. There are 5 men in the line at time  $t = 0$ .

- How many men enter the line for the cableway during the time interval  $0 \leq t \leq 150$ ?
- During the time interval  $0 \leq t \leq 150$ , there are always men in line for the cableway. How many men are in line at  $t = 150$ ?
- For  $t > 150$ , what is the first time  $t$  that there are no men in line for the cableway?
- For  $0 \leq t \leq 150$ , at what time  $t$  is the number of men in line a minimum? Find the number of men in line when the number of men in line is minimized? Round your answer to the nearest number of men.

2. A particle moves along the  $x$ -axis so that its velocity at time  $t$  is given by  $v(t) = -(t + 1)\cos\left(\frac{t^2}{7}\right)$ . At time  $t = 0$ , the particle is at position  $x = 2$ .
- What is the acceleration of the particle at time  $t = 3$ ? Is the speed of the particle increasing at  $t = 3$ ? Why or why not?
  - Find all times  $t$  in the open interval  $0 < t < 4$  when the particle changes direction.
  - What is the total distance traveled by the particle from time  $t = 0$  until time  $t = 4$ ?
  - During the time interval  $0 \leq t \leq 4$ , what is the greatest distance between the particle and the origin? Show your work.

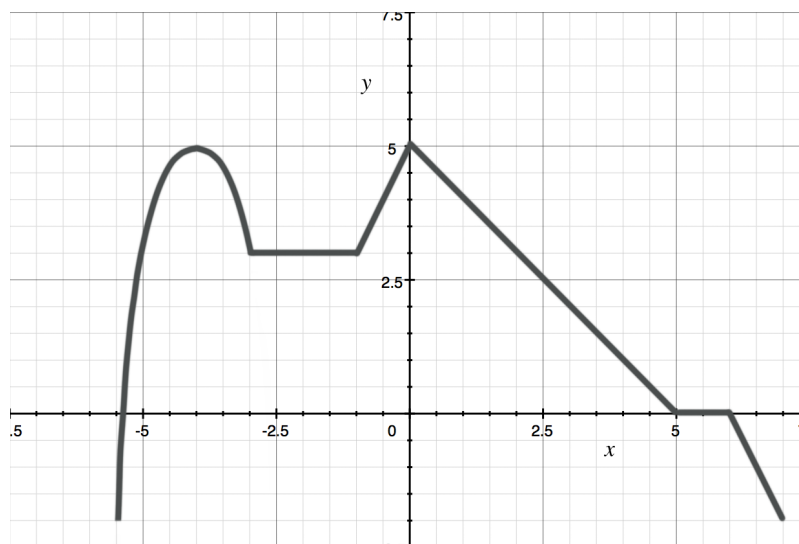
**END OF PART A, SECTION II**

**SECTION II, PART B**

**Time - 60 Minutes**

**Number of problems - 4**

**NO CALCULATOR IS ALLOWED FOR THESE PROBLEMS.**



3. The graph of the continuous function  $h$ , the derivative of the function  $f$ , is shown above. The function  $h$  is  $h(x) = -2(x + 4)^2 + 5$  for  $-6 \leq x \leq -3$  and piecewise linear for  $-3 \leq x \leq 7$ .

a. If  $f(-1) = 3$ , what is the value of  $f(7)$ ?

b. Evaluate  $\int_{-6}^{-1} g(x) dx$ .

c. For  $-6 < x < 7$ , on what open intervals, if any, is the graph of  $f$  both decreasing and concave down? Give a reason for your answer.

d. Find the  $x$ -coordinate of each point of inflection of the graph of  $f$ . Give a reason for your answer.

Distance $x$ (cm)	0	2	3	5	8	10
Temperature $T(x)$ (°C)	100	90	81	73	60	51

4. A metallic wire of length 10 cm is heated at one of its ends. The table above gives selected values of the temperature of the wire  $x$  cm from the heated end. The function  $T$  is decreasing and twice differentiable.
- Determine  $T'(9)$ . Show your work.
  - Write an integral expansion in terms of  $T(x)$  for the average temperature of the wire. Determine the average temperature of the wire using a trapezoidal sum with five subintervals shown by the data in the table.
  - Find  $\int_0^{10} T'(x) dx$ . What is the meaning of  $\int_0^{10} T'(x) dx$  in terms of the temperature of the wire.
  - Explain why there must be at least one distance  $x$ , for  $0 < x < 10$ , such that  $T'(x) = -4$ .

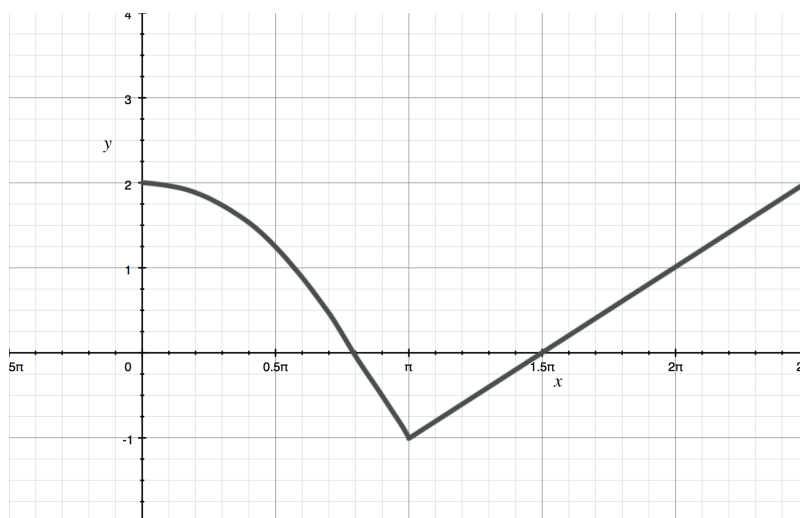
5. Let  $f$  be the function defined by  $f(x) = e^{3x} \sin 3x$ .

a. What is the average rate of change of  $f$  on the interval  $\frac{\pi}{2} < x < 2\pi$ ?

b. What is the slope of the line tangent to the graph of  $f$  at  $x = \pi$ ?

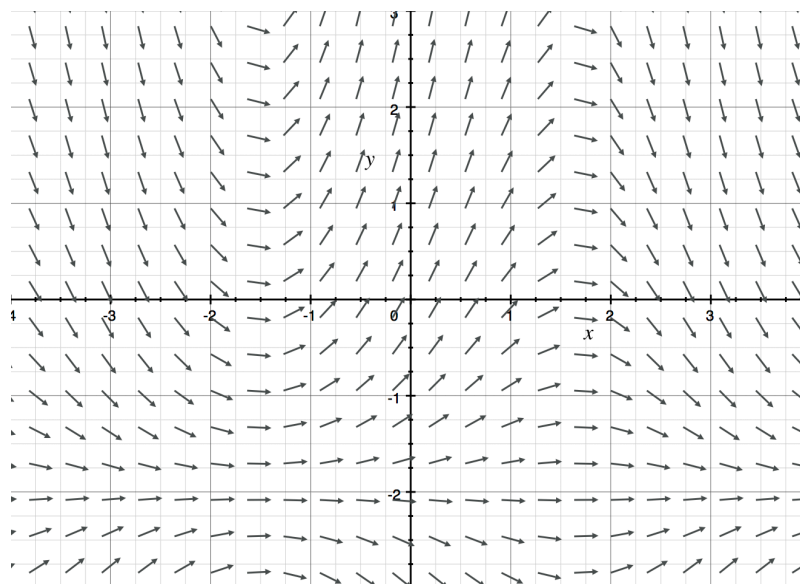
c. What is the absolute maximum value of  $f$  on the interval  $0 \leq x \leq \pi$ ?

d. Let  $g$  be a differentiable function such that  $g(\pi) = 0$ . The graph of  $g'$ , the derivative of  $g$  is shown below. Find the value of  $\lim_{x \rightarrow \pi} \frac{g(x)}{f(x)}$  or state that it does not exist.



6. Consider the differential equation  $\frac{dy}{dx} = (y + 2)\cos x$ . Let  $y = f(x)$  be the particular solution to the differential equation with the initial condition  $f(0) = 0$ . The function  $f$  is defined for all real numbers.

a. A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point  $(0,0)$ .



- b. Write an equation for the line tangent to the solution curve in part *a* at the point  $(0,0)$ . Use the equation to approximate  $f(0.5)$ .
- c. Find  $y = f(x)$ , the particular solution to the differential equation with the initial condition  $f(0) = 0$ .

**STOP**

**END OF EXAM**